Homework 3 - Solutions

Double Integrals

① $\int_0^3 \int_0^1 (16 - x^2 - 3y^2)\,dy\,dx$

$= \int_0^3 \left[16y - x^2 y - y^3 \right]\Big|_0^1 \, dx$

$= \int_0^3 \left[(16 - x^2 - 1) - (0 - 0 - 0) \right] dx$

$= \int_0^3 (15 - x^2)\,dx$

$= \left(15x - \frac{1}{3}x^3 \right)\Big|_0^3$

$= \left[(45 - 9) - (0 - 0) \right] = \boxed{36}$

Sequences

① $\lim_{n\to\infty} \frac{7 + 15n^4}{137 - 22n^3 + 47n^4} = \frac{15}{47}$ $\boxed{\text{converges to } \frac{15}{47}}$

② $\lim_{n\to\infty} \frac{3^{n+4}}{5^n} = \lim_{n\to\infty} 3^4 \cdot \left(\frac{3}{5}\right)^n = 0$ $\boxed{\text{converges to } 0}$

since $\frac{3}{5} < 1$

(3) $\lim_{n\to\infty} \sqrt{\frac{n-14}{7n+1}} = \sqrt{\lim_{n\to\infty}\left(\frac{n-14}{7n+1}\right)} = \sqrt{\frac{1}{7}}$ $\boxed{\text{Converges to } \sqrt{\tfrac{1}{7}}}$

(4) $\lim_{n\to\infty} ne^{-n} = \lim_{n\to\infty} \frac{n}{e^n}$

$= \lim_{n\to\infty} \frac{1}{e^n}$ by L'Hopital's Rule

$= 0$ $\boxed{\text{Converges to } 0}$

(5) $\left\{\frac{1}{1}, \frac{1}{3}, \frac{1}{2}, \frac{1}{4}, \frac{1}{3}, \frac{1}{5}, \frac{1}{4}, \frac{1}{6}, \dots\right\}$

this sequence can be written as

$$a_n = \begin{cases} \frac{2}{n+1} & \text{if } n \text{ odd} \\ \frac{2}{n+4} & \text{if } n \text{ even} \end{cases}$$

then $\frac{2}{n+4} \leq a_n \leq \frac{2}{n+1}$

$$0 = \lim_{n\to\infty} \frac{2}{n+4} \leq \lim_{n\to\infty} a_n \leq \lim_{n\to\infty} \frac{2}{n+1} = 0$$

Then by the Squeeze Thm, $\lim_{n\to\infty} a_n = 0$

$\boxed{\text{Converges to } 0}$

⑥ $a_n = \frac{1+2+3+\dots+(n-1)}{n!} = \frac{\frac{n(n-1)}{2}}{n!} = \frac{n(n-1)}{2n!} = \frac{1}{2(n-2)!}$

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1}{2(n-2)!} = 0$

$\boxed{\text{converges to } 0.}$

⑦ $\lim_{n\to\infty} \frac{\cos(n)}{n^2}$

for all $n>0$ $\quad -1 \le \cos(n) \le 1$

So $\quad \frac{-1}{n^2} \le \frac{\cos(n)}{n^2} \le \frac{1}{n^2}$

Since
$0 = \lim_{n\to\infty} \frac{-1}{n^2} \le \lim_{n\to\infty} \frac{\cos(n)}{n^2} \le \lim_{n\to\infty} \frac{1}{n^2} = 0$

then by the Squeeze Thm,

$\lim_{n\to\infty} \frac{\cos(n)}{n^2} = 0$ $\quad \boxed{\text{Converges to } 0}$

⑧ $\lim_{n\to\infty} \frac{2+3^n}{2+3^{n+1}} = \lim_{n\to\infty} \frac{3^n}{3^{n+1}} = \lim_{n\to\infty} \frac{3^n}{3\cdot 3^n} = \frac{1}{3}$

$\boxed{\text{Converges to } 1/3}$